Alpha Chain Structures of ^{12}C

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N-α structures of light nuclei with axial symmetry are studied using relativistic Hartree approximation. Metastable excited states are searched in a configuration space which allows linear alpha chain structures. As a result, it is shown that ^{12}C has ^{8}Be+α resonance state at about 1 MeV above ^{8}Be–α threshold as an asymmetric 3-α linear-chain structure, which plays an important role in stellar nucleosynthesis.

Hydrogen burning and helium burning are very important processes in the evolution of young stars, playing roles as the nuclear synthesizer for light nuclei and as the source of energy production. An interesting feature appearing in the helium burning process is the possible N-α structures of light nuclei. Since the even-even light nuclei, such as ^{4}He, ^{8}Be, ^{12}C, ^{16}O, and ^{20}Ne, have similar binding energies per nucleon but larger than that of an alpha particle, the ^{4}He nucleus in its ground state, we may assume that these nuclei are composed of alpha particles at least for some features of these nuclei. Öpik [1] and Salpeter [2] pointed out that unstable ^{8}Be can capture additional α particle to form ^{12}C within their life time in a star, which is composed of ^{4}He as the ash of its hydrogen burn. Most of ^{12}C in our universe may be produced by ^{8}Be+$\alpha \leftrightarrow ^{12}$C reaction. Hoyle suggested [3] that this reaction must proceed through a hypothetical 0^+ resonance state of ^{12}C at energy of 0.4 MeV above 3-α threshold. The second 0^+ excited state of ^{12}C was found with the resonance energy of $E_r = 0.3796$ MeV above the threshold (0.278 MeV above ^{8}Be+α threshold) and the full width of $\Gamma = 8.5 \times 10^{-6}$ MeV [4,5].

In the N-α structure model, ^{12}C is considered to be made up of three alpha particles bound to each other. Three alpha particles form an equilateral triangle in its ground state [6] and form a linear chain for the 0^+ excited state. Alpha structures of the low-lying states of ^{12}C and ^{16}O, including their 0^+ excited states, have been studied within a framework of nonrelativistic macroscopic and microscopic methods [6–14]. These studies treat alpha particles as inert clusters, and thus miss any detailed structure of nuclei and any change of the internal structure of alpha particles in a nucleus. For the study of detailed nuclear structure in these highly excited super-deformed nuclei, we need to use a mean field approach.

In nuclear mean field theory, a nuclear system is composed of nucleons interacting strongly through a mean field potential. While the nonrelativistic mean field approach uses a phenomenological mean field potential as a function of nucleon density, the mean field potential in relativistic mean field theory is determined as a resultant of meson exchange. After Walecka proposed a relativistic mean field (RMF) approach for a nuclear system in which Dirac nucleons interact by exchanging classical meson fields [15], it has been extended to include various quantum effects and successfully applied in describing nuclear matter and finite nuclei [15–27]. However the usage of the mean field approach is limited to spherical or moderately deformed nuclei due to the limitation of the size of the functional basis space. To describe highly deformed alpha chain structure, we construct, in this paper, a functional basis space for the axially symmetric RMF calculation by appropriately locating single particle levels of ^{4}He.

In the relativistic mean field approach, nucleon-nucleon nonlocal interactions are replaced by nucleon-meson local interactions [16] of various meson exchange. For the long range attractive and short range repulsive characteristics of nucleon-nucleon interaction, we at least include in RMF an isoscalar scalar meson field (σ meson) and a massive isoscalar vector meson field (ω meson). An isovector vector meson field (ρ meson) is included to handle the charge exchange and the electromagnetic field for the electromagnetic interaction among protons. We also include the nonlinear self-interaction terms of the scalar meson field to improve the compressibility of nuclear matter and the deformation of finite nuclei. By choosing carefully the coupling constants and the meson masses as the parameters, we can get quantitative agreement with experiments for spherical and deformed nuclei [16–22]. Lee *et al.* studied deformed nuclei within the RMF with various parameter sets [17]. We have used the same numerical methods and parameter sets for our calculation here.

For an α particle, linear parameter sets L1, L2, and L3 give too little binding energy (15.4 MeV $\sim$ 3.6 MeV) while the nonlinear parameter set NL1 gives about the right binding energy (31.9 MeV) compared to the empirical value of 28.27 MeV [4]. On the other hand, for the ground states of ^{12}C and ^{16}O, NL1 gives much stronger binding (136.1

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MeV and 206.5 MeV respectively) while L1 gives 94.1 MeV and 128.9 MeV compared to the empirical values of 92.2 MeV and 127.6 MeV [28]. These results indicate that we need to search for a better parameter set which is good for both ^{4}He and ^{12}C to study the alpha structure of ^{12}C. However, for simplicity, we used parameter set L1 in this paper since we are interested more in the existence of the alpha-chain structure of ^{12}C within a self-consistent mean field theory rather than the detailed properties.

The numerical method is exactly the same as in Ref. [17] except the methods of how to set the functional basis space and the initial configuration of nuclei for the self-consistent calculation. Initially, ^{4}He nuclei in its ground state are linearly arranged on the z-axis and each ^{4}He nucleus is accompanied by several excited single nucleon levels of ^{4}He forming a large enough functional basis space to handle highly deformed alpha-chain structure.

Starting with two α particles placed 4 fm to 13 fm apart, nucleons in the 2-α system reassembled to form the ground state of ^{8}Be, with the binding energy of 43.1 MeV (empirical binding energy is 56.5 MeV). This state has a peanut shaped prolate deformation with 2.8 fm separation between the supposed two alpha particles. This state is about 13 MeV lower than the two-alpha threshold in contrast to the empirical value which shows that the ground state of ^{8}Be has almost the same energy as the two-alpha threshold, thus making ^{8}Be unstable. Remember here that the L1 parameter set produces too little binding for small nuclei such as He and Be, the worse for the smaller nuclei. When the initial separation of alpha particles was above 13 fm, we got an excited oblate state of binding energy 34.5 MeV for the L1 set. These converged solutions are just the same as the original method of Ref. [17] calculated without any consideration of alpha structure.

Starting with three α particles linearly arranged with separations of 4 fm to 10 fm, three α particles combined into the ground state of ^{12}C with binding energy of 94.0 MeV. For calculations with initial separations above 10 fm, we got continuum states in which two α particles combined together to form the ground state of ^{8}Be and the remaining α particle was separated away from the ^{8}Be on its symmetry axis. Both the ^{8}Be piece and the ^{4}He piece of this system have the same binding energies, rms radii, and quadrupole moments as the ground state of ^{8}Be and the ground state of ^{4}He respectively. The continuum state solutions show that ^{12}C has a fission barrier of α emission at around 10 fm separation. The energy of this ^{8}Be–^{4}He system is highest when the separation is about 10 fm and the energy is lowered toward the ^{8}Be+α threshold as the separation becomes larger. At around 10 fm separation the energy is -57.2 MeV which is 1.3 MeV above the ^{8}Be+α threshold (-58.5 MeV for L1 parameter). Although the energy is much higher than the empirical value (-84.54 MeV) for the second 0^+ state of ^{12}C, we may relate this continuum region to the 0_2^+ excited state. Experimentally, the 0_2^+ excited state of ^{12}C is 0.38 MeV above the three-alpha threshold and 0.28 MeV above the ^{8}Be+α threshold [4,5]. Remembering that the L1 parameter set gives too small binding energy for light nuclei and good fit for ^{12}C and ^{16}O, this ^{8}Be–^{4}He system might actually be the ^{8}Be+α resonance which is an asymmetric three-α linear chain state in the alpha cluster model. This state is not a three-alpha resonance state having equally spaced α particles. In these calculations no three-alpha resonance state was found.

We have also tested this resonance state starting with initial states built with a ground state ^{8}Be and a ground state ^{4}He placed on the symmetry axis of ^{8}Be. When the initial separation is 4 fm to 10 fm apart, the system converged to the ground state of ^{12}C and the system becomes a ^{8}Be+α resonance state for the initial separation of 10 fm to 20 fm. We, within our calculations, have found no resonance state of the excited oblate ^{8}Be and α which may correspond to an isosceles triangular configuration in an alpha cluster model. Even if we started with the initial configuration built with the excited oblate ^{8}Be and an alpha, the calculation converged to the resonance state of the ground state ^{8}Be and an alpha particle for any initial separation.

In this paper, we have studied the alpha structure of light nuclei in a relativistic mean field approach. As a result, we have seen that the ground state of ^{8}Be may be considered as two alpha clusters separated 2.8 fm apart with some overlap of surface region. We have also seen that ^{12}C has a continuum state region which might be related to a ^{8}Be+α resonance state as the 0^+ state which is empirically at 0.38 MeV above the three α threshold and that there is no 3-α resonance state. These results confirm that ^{12}C might be produced through ^{8}Be+α resonance state in a star rather than a direct combination of three alpha particles through a 3-α resonance. To check if the continuum state actually corresponds to a resonance state, we need to find resonance width or level density by extending our method or combining with other method. We may apply this calculation in study of α structure in ^{16}O, ^{20}Ne, and ^{24}Mg. For the study of more detailed structure, we should first find a better parameter set which is good for the mass range of 4 to 30.

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